Classical Aspects of Accelerated Unruh-DeWitt Type Monopole Detectors

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We have shown the classical correspondence of Unruh effect in the classical relativistic electron theory in our previous work [1]. Here we demonstrate the analogy between the classical relativistic electron theory and the classical Unruh-DeWitt type monopole detector theory. The field configuration generated by a uniformly accelerated detector is worked out. The classical correspondence of Unruh effect for scalar fields is shown by calculating the modified energy density for the scalar field around the detector. We conclude that a classical monopole detector cannot find any evidence about its acceleration unless it has a finite size.

I. INTRODUCTION

A uniformly accelerated testing particle in Minkowski vacuum might observe a background thermal radiation, with the temperature

$$T_U = \frac{\hbar a}{2\pi k_B c} \tag{1}$$

proportional to the acceleration a of the testing particle. This remarkable phenomenon, the Unruh effect [2,3], has been considered as a connection between quantum($\hbar$), statistical(k_B) and relativistic physics. However, since the quantum correlation functions for free fields are Green's functions of the corresponding classical field theories, if there exists thermal characters in a correlation function or a vacuum expectation value of the stress-energy tensor, corresponding information should be found in its classical theory [4–6]. Indeed, in our previous work [1], we explicitly showed a classical correspondence of Unruh effect in classical relativistic electron theory: the vacuum expectation value of the energy density for a point-like electron is identical to its classical self-energy density. In the classical, point-like framework, the thermal interpretation seems to be un-necessary and the Unruh temperature (1) is an artifact by identifying the power spectrum to the Planckian-like ones.

Since the prototype of the detector theory in Rindler space concerns a point-like monopole detector linearly coupled with scalar fields [3,7], we would like to study such kind of models in the following and try to reach the same conclusion. In Section II we recall the classical electron theory and the solution of electromagnetic(EM) field generated by a uniformly accelerated charge. Then a model of classical monopole detectors for massless scalar fields is given in Section III for comparing with the literature about the Unruh effect. Finally we have some discussions in Section IV.

II. UNIFORMLY ACCELERATED ELECTRIC CHARGE

To begin with, let us consider the action for the relativistic Lorentz electron [8]

$$S = -m \int d\tau \sqrt{-\frac{dz_\mu}{d\tau}\frac{dz^\mu}{d\tau}} - \int d^4x \sqrt{-g}\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$
$$+ \int d\tau d^4x \sqrt{-g} j_\mu(x,\tau) A^\mu(x), \tag{2}$$

where $F_{\mu\nu} \equiv D_\mu A_\nu - D_\nu A_\mu$ and the current j_μ is defined by

$$j_\mu \equiv e\frac{dz_\mu}{d\tau}\delta^4(x - z(\tau)) \tag{3}$$

with the coupling constant e. The solutions for vector fields can be written in $A^\mu = A^\mu_{\text{in}} + A^\mu_{\text{ret}}$ or $A^\mu = A^\mu_{\text{out}} + A^\mu_{\text{adv}}$, depending on the choice of boundary conditions. Here A^μ_{ret} and A^μ_{adv} are the retarded and advanced fields respectively. Let

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$$\bar{A}^{\mu} = \frac{1}{2}(A_{\text{in}}^{\mu} + A_{\text{out}}^{\mu}) \tag{4}$$

$$A_{\pm}^{\mu} = \frac{1}{2}(A_{\text{ret}}^{\mu} \pm A_{\text{adv}}^{\mu}) \tag{5}$$

such that $\bar{A}^{\mu} = A_{\text{in}}^{\mu} + A_{-}^{\mu}$ and $A^{\mu} = \bar{A}^{\mu} + A_{+}^{\mu}$. It is known that the sigular behavior of the solution are all present in A_{+}^{μ}, hence one re-write

$$S_{\text{ren}} = -m \int d\tau \sqrt{-\frac{dz_{\mu}}{d\tau}\frac{dz^{\mu}}{d\tau}} + \int d\tau d^4 x \sqrt{-g} j_{\mu}(x,\tau)\bar{A}^{\mu}(x)$$
$$- \int d^4 x \sqrt{-g}\frac{1}{4}\left(\bar{F}_{\mu\nu}\bar{F}^{\mu\nu} + 2\bar{F}_{\mu\nu}F_{+}^{\mu\nu}\right) \tag{6}$$

by dropping all of the divergent terms. Variation on z^{μ}, $\bar{A}^{\mu}$ and A_{+}^{μ} then gives Lorentz-Dirac equation as well as Maxwell equations,

$$ma^{\mu}(\tau) = e\bar{F}^{\mu\nu}v_{\nu}(\tau), \tag{7}$$
$$\partial_{\mu}\left[\bar{F}^{\mu\nu}(x) + F_{+}^{\mu\nu}(x)\right] = -j^{\mu}(x), \tag{8}$$
$$\partial_{\mu}\bar{F}^{\mu\nu}(x) = 0, \tag{9}$$

where $v^{\mu} \equiv dz^{\mu}/d\tau$ is the proper velocity and $a^{\mu} \equiv dv^{\mu}/d\tau$ is the proper acceleration.

An electric charge accelerated in a constant proper acceleration $|a_{\mu}a^{\mu}|^{1/2} = a$ would go along the hyperbolic trajectory $z'^{\mu} = (a^{-1}\sinh a\tau, a^{-1}\cosh a\tau, 0, 0)$, parameterized by its proper time τ.[1] The corresponding EM field was firstly given by Born [9] in 1909, then in 1955 Bondi and Gold [10] found a more general solution,

$$E_z = F^{tz} = -\frac{4e}{a^2 X^3}\left(a^{-2} + t^2 - z^2 + \rho^2\right)\theta(z+t), \tag{10}$$

$$E_{\rho} = F^{t\rho} = \frac{8e\rho z}{a^2 X^3}\theta(z+t) + \frac{2e\rho}{\rho^2 + a^{-2}}\delta(z+t), \tag{11}$$

$$B_{\phi} = F^{z\rho} = \frac{8e\rho t}{a^2 X^3}\theta(z+t) - \frac{2e\rho}{\rho^2 + a^{-2}}\delta(z+t), \tag{12}$$

where

$$X \equiv \sqrt{4a^{-2}\rho^2 + \left(a^{-2} + t^2 - z^2 - \rho^2\right)^2}, \tag{13}$$

and the step function $\theta(x)$ is defined by

$$\theta(x) = \begin{cases} 1 & \text{for } x > 0 \\ 1/2 & \text{for } x = 0 \\ 0 & \text{for } x < 0 \end{cases}. \tag{14}$$

The improvement is that Bondi-Gold solution satisfies Maxwell equations in the whole spacetime including the event horizon.

In our previous paper [1], we noted that the classical energy density measured by the co-moving observer at $z^{\mu} = z'^{\mu} + (0, 0, \rho, 0)$ can be related to the Unruh effect, by letting the correlation $\rho = 2a^{-1}\sinh(a\Delta/4)$ then performing a Fourier transformation with respect to Δ. Below we illustrate the same relation in the classical version of the original detector model for scalar fields.

III. UNRUH-DEWITT TYPE MONOPOLE DETECTOR

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[1]For linearly accelerated detectors, we use the cylindrical coordinate $ds^2 = -dt^2 + dz^2 + d\rho^2 + \rho^2 d\theta^2$ with $c = 1$.

A. the model

An Unruh-DeWitt type monopole detector model could be formulated by the action

$$S = S_q + S_\phi + S_{\text{int}}, \tag{15}$$

where

$$S_\phi = -\int d^4x \sqrt{-g} \frac{1}{2} \partial_\mu \phi \partial^\mu \phi, \tag{16}$$

is the action for a massless scalar field ϕ,

$$S_q = \int d\tau \left[\frac{1}{2} (\partial_\tau q)^2 - V(q) \right], \tag{17}$$

is the action for the detector with monopole moment q, and

$$S_{\text{int}} = e \int d\tau d^4x \sqrt{-g} q(\tau) \phi(x) \delta^4(x - x'(\tau)), \tag{18}$$

is the interaction-at-a-point with coupling constant e. In this paper we choose $V = \omega^2 q^2/2$ as a harmonic oscillator for simplicity. Now the similarity between the Unruh-DeWitt type monopole detector and the classical electron theory Eq.(2) is obvious. The main difference is that we put the monopole moment here as an extra degree of freedom to the motion, while the dipole moment in electron theory (2) is directly related to the position of the electron. Hence the trajectory of the monopole detector can be arbitrary in our model.

To obtain a renormalized action, one can decompose ϕ in the same fashion of the classical electron theory, namely,

$$\phi = \bar{\phi} + \phi_+, \tag{19}$$

where $\bar{\phi}$ and ϕ_+ are homogeneous and inhomogeneous solutions of ϕ respectively. Then, omitting the singular terms, the regular part reads

$$\begin{aligned}
S_{\text{ren}} = &\int d\tau \left[\frac{1}{2} (\partial_\tau q)^2 - V(q) \right] \\
&- \int d^4x \sqrt{-g} \frac{1}{2} \left[\partial_\mu \bar{\phi} \partial^\mu \bar{\phi} + 2 \partial_\mu \phi_+ \partial^\mu \bar{\phi} \right] \\
&+ e \int d\tau d^4x \sqrt{-g} q(\tau) \bar{\phi}(x) \delta^4(x - x'(\tau)),
\end{aligned} \tag{20}$$

which yields the equation of motion and field equations as follows,

$$\partial_\tau^2 q + V'(q) = e\bar{\phi}(x'(\tau)), \tag{21}$$

$$\Box(\bar{\phi} + \phi_+) = e \int d\tau q(\tau) \delta^4 (x - x'(\tau)), \tag{22}$$

$$\Box \bar{\phi} = 0, \tag{23}$$

where $V'(q) \equiv \delta V/\delta q = \omega^2 q$. The first equation is equivalent to the equation of motion for a driven oscillator. The second equation descibes the field configuration generated by a point source with a time-varying charge. The third is the homogeneous equation for the scalar field.

B. static case in Minkowski space

Suppose the detector locates at the origin, namely, $x'(\tau) = (t, 0, 0, 0)$. In Minkowski space, the static solution requires that $\tau = t$, $q(\tau) = q_0$ and $\bar{\phi}(x'(\tau)) = \phi_0$, where q_0 and ϕ_0 are constants of t. Then Eq.(21) gives $\omega^2 q_0 = e\phi_0$ and Eq.(22) gives

$$\phi_+(x) = eq_0 \int d\tau G_+(x - x'(\tau)) = \frac{eq_0}{8\pi r} \tag{24}$$

such that

$$\phi = \frac{e^2\phi_0}{8\pi\omega^2 r} + \bar{\phi}\Big|_{\bar{\phi}(x'(\tau))=\phi_0} \tag{25}$$

where $r \equiv \sqrt{\rho^2 + z^2}$ is the radius in spherical coordinate and $G_+(x - x') \equiv (4\pi)^{-1}\theta(t - t'(\tau))\delta(|x - x'(\tau)|^2)$ is the retarded Green's function of ϕ_+. Since the solution above has the time-reversal symmetry, the retarded field starts at $t = -\Delta/2$ is equal to the advanced field ends at $t = +\Delta/2$. Then one can introduce the correlation $r = |\pm\Delta/2|$ and write

$$\phi_+(-\Delta/2)\phi_+(\Delta/2) = \frac{e^2 q_0^2}{16\pi^2\Delta^2}. \tag{26}$$

On the other hand, given the Hadamard's elementary function for massless scalar field, $D^{(1)}(x - x'(\tau)) = \langle 0|\{\phi(x)\phi(x')\}|0\rangle = \hbar/2\pi^2(x^\mu - x'^\mu(\tau))(x_\mu - x'_\mu(\tau))$ [7], one has

$$\langle 0|\{\phi(t = -\Delta/2)\phi(t' = \Delta/2)\}|0\rangle = -\frac{\hbar}{2\pi^2\Delta^2}, \tag{27}$$

whose Δ-dependence is the same as Eq.(26).

One may further compare physical quantities such as energy densities from both sides. The modified stress-energy tensor in Minkowski space is defined by [7]

$$T_{\mu\nu} = (1 - 2\xi)\,\phi_{,\mu}\phi_{,\nu} - 2\xi\phi\phi_{;\mu\nu}$$
$$+ \left(2\xi - \frac{1}{2}\right)g_{\mu\nu}g^{\rho\sigma}\phi_{,\rho}\phi_{,\sigma} + \frac{\xi}{2}g_{\mu\nu}\phi\Box\phi, \tag{28}$$

where ξ is a constant parameter. Here T_{tt} with different values of ξ are different by total derivatives of space, hence their local physics are the same [11]. Now a straightforward calculation gives the classical energy density

$$T_{tt}(t, r) = \frac{e^2 q_0^2(1 - 4\xi)}{128\pi^2 r^4} = \frac{e^2 q_0^2(1 - 4\xi)}{8\pi^2\Delta^4}, \tag{29}$$

at the point $(t, r, 0, 0)$ in spherical coordinate, while the expectation value of the energy density of the detector reads

$$\langle T_{tt}\rangle = \frac{\alpha'}{2}\lim_{x^\mu \to x'^\mu}\left[\frac{1}{2}\partial_t\partial_{t'} - \left(2\xi - \frac{1}{2}\right)(\partial_x\partial_{x'} + \partial_y\partial_{y'} + \partial_z\partial_{z'})\right.$$
$$\left. - \frac{3\xi}{2}\partial_t^2 - \frac{\xi}{2}(\partial_x^2 + \partial_y^2 + \partial_z^2)\right]D^{(1)}(x, x')$$
$$= \lim_{\Delta \to 0}\frac{3\alpha'\hbar}{2\pi^2\Delta^4}. \tag{30}$$

As suggested by Mane [5], the vacuum power flux for the EM field quantized in rotating frame [4] can be identified to the classical synchrotron radiation from a rotating electron, by multiplying an overall factor proportional to the EM fine-structure constant $\alpha = e^2/\hbar c$ to the former. This factor, originated from the coupling between electrons and photons, is natural in quantum electrodynamics, but not so obvious in the semi-classical EM theory with point-like sources. The same identification is confirmed by Hirayama and Hara [6] and works well in our previous article [1]. Hence we recognize that above quantum expectation value is actually to the classical order. To identify both results, we simply compare Eq.(30) with Eq.(29) and choose the overall factor $\alpha' = e^2 q_0^2(1 - 4\xi)/12\hbar$.

C. acceleration and Planck factor

Next let us consider the detector moving in a uniform acceleration a. Again the trajectory is $x'(\tau) = (a^{-1}\sinh a\tau, a^{-1}\cosh a\tau, 0, 0)$. Suppose the detector is in equilibrium with the background, which corresponds the classical solutions static with respect to τ. Requiring constant solutions that $q(\tau) = q_0$ and $\bar{\phi}(x'(\tau)) = \phi_0$ along the trajectory, where q_0 and ϕ_0 are constants of τ subject to $\omega^2 q_0 = e\phi_0$ from Eq.(21), the total solution of ϕ then reads

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$$\phi = \frac{e^2\phi_0}{4\pi\omega^2 aX}\theta(z+t) + \bar{\phi}\Big|_{\bar{\phi}(x'(\tau))=\phi_0} \tag{31}$$

where X has the same definition as Eq.(13).

In the region with $z+t > 0$ and $z-t > 0$, (31) has the time-reversal symmetry, which makes the retarded field equal to the advanced field like the static case in Minkowski space (clearly this property is true only when the acceleration is uniform). At $z^\mu = (0, a^{-1}, \rho, 0)$, the inhomogeneous part of ϕ has

$$\phi_+(-\Delta/2)\phi_+(\Delta/2) = \left(\frac{eq_0}{4\pi a}\right)^2 \frac{1}{\rho^2(\rho^2 + 4a^{-2})} = \frac{e^2 q_0^2 a^2}{16\pi^2 \sinh^2(a\Delta/2)} \tag{32}$$

if one substitute the correlation $\rho = 2a^{-1}\sinh(a\Delta/4)$ given in Ref. [1]. This is identical to the correlation function for this case in terms of Hadamard's elementary function up to an overall factor, hence acquires the same thermal character.

Recall that the first order perturbation theory of a quantum detector with the same interacting action S_{int} (unrenormalizd, Eq.(18)) has the transition probability per unit proper time [3,7],

$$P = \frac{e^2}{2\pi}\sum_{E\neq E_0}|\langle E|q(0)|E_0\rangle|^2 \int_{-\infty}^{\infty} d(\Delta\tau)e^{-i(E-E_0)(\Delta\tau)}D^{(1)}(\Delta\tau)$$
$$= \frac{e^2}{2\pi}\sum_{E\neq E_0}\frac{(E-E_0)|\langle E|q(0)|E_0\rangle|^2}{e^{2\pi(E-E_0)/a}-1} + \text{singular terms}, \tag{33}$$

if the detector was prepared in its ground state at past null infinity ($\tau \to -\infty$). It is clear that the Planck factor in P is totally from the regular part of the correlation function $D^{(1)} \sim 1/\sinh^2(a(\Delta\tau)/2)$ in $\Delta\tau \to 0$ limit. Nevertheless, in our classical theory, no transition rate can be defined for the detector at all. To see the thermal character of the classical uniformly accelerated detector, one has to look into the energy density. At $z^\mu = (0, a^{-1}, \rho, 0)$, the energy density for the inhomogeneous part of ϕ reads

$$T_{tt}(\phi_+) = \frac{e^2 q_0^2 \left[1 - 4\xi + (1-6\xi)a^2\rho^2\right]}{8\pi^2 \rho^4(4+a^2\rho^2)^2}, \tag{34}$$

while the quantum expectation value of T_{tt} is

$$\langle T_{tt}\rangle = \frac{\alpha'\hbar a^4}{32\pi^2}\lim_{\tau_\pm \equiv \pm\Delta/2 \to 0}\frac{1 + 2\cosh a(\tau_+ + \tau_-)}{\sinh^4[a(\tau_+ - \tau_-)/2]}$$
$$= \lim_{\Delta\to 0}\frac{e^2 q_0^2 a^4(1-4\xi)}{128\pi^2 \sinh^4(a\Delta/2)}. \tag{35}$$

After one substitutes $\rho = 2a^{-1}\sinh(a\Delta/4)$ into Eq.(34), above two energy densities are exactly the same when $\xi = 1/6$, which corresponds to a conformal invariant scalar field theory. Furthermore, the Δ-dependence in Eq.(35) is identical to those for classical uniformly accelerated electric charges (see Eq.(13)-(16) in Ref. [1]), hence one can apply the same approach in renormalization and find the "vacuum energy" being $11e^2 q_0^2 a^4/17280\pi^2$ for $\xi = 1/6$. Again the Planck factor can be obtained, for each value of ξ, by performing a Fourier transformation with respect to Δ on the classical energy density (34).

IV. DISCUSSION

A. existence of specific motion

For our Unruh-DeWitt type monopole detector theory, the accelerating motion of the detector is actually arbitrary. Nevertheless, the same arbitrariness is not true in detector theories for EM or gravitational fields. In the EM case, the dipole moment coupled to the homogeneous EM field $\bar{A}^\mu$ is directly related to the position of the charge, rather than a degree of freedom independent of motion in our monopole model. Whether there exists a specific accelerating motion depends on the existence of this solution of the Lorentz-Dirac equation together with Maxwell equations. Stationary

circular motion around the nucleus, for example, is impossible for a classical electric charge [12], hence the vacuum state strictly defined by the corresponding coordinate system has no physical meaning for a point-like electric charge.

But one still has the right to consider the instantaneous "vacuum stress-energy" for any accelerated detector, as we pointed out in our previous article [1]. The reason is that the instantaneous "vacuum stress-energy" can be locally defined by the co-moving observer, while the power spectrum needs the knowledge about the whole history to perform the Fourier transformation.

B. particle detector creates particle

There might exist more than one timelike Killing vectors in some spacetimes. Along these time axes, one can formulate different Hamiltonian theories, whose vacua might disagree with each other. In a pure field theory consideration, there is no guideline to determine which vacuum is correct, unless one imposes a detector or a testing particle, fixes physical boundary conditions, then observes the response of the detector.

So far what one can do in testing the detector theory is also to accelerate some elementary particles without structure, such as electrons, then measure their responses in laboratories [13]. Therefore only the back reaction of the detector is interesting and measurable for experimental physicists, yet whether the detector really see a thermal bath is not important here.

When the back reaction of the detector on the field configuration is taken into account, an classical observer co-moving with the uniformly accelerated detector may conclude that the detector experiences a thermal bath of Rindler particles rather than a zero-temperature vacuum, by measuring the static field strength around the detector then introducing the correlation between their distance (ρ) and the clock of the detector (τ or Δ). Hence the particle population that the detector experiences, speculated by the co-moving observer, depends on how the detector (and the co-moving observer) moves. In this sense the particle detector itself is the creator of the particle.

C. Can a classical point-like monopole detector detect its acceleration?

No matter how close the co-moving observer is to the point-like detector, they would never overlap by definition. While the self-energy density for the scalar field is singular right at the position of the detector, the internal state of the classical monopole detector is actually finite and static with respect to its proper time. The constant solution for monopole moment $q = q_0$ corresponds to the lowest "effective potential energy" $V - \mathcal{L}_{\text{int}} = \omega^2 q_0^2/2 - e\phi_0 q_0$ for the detector. Since it does not exert work, it seems that one can relate this "effective potential energy" to the entropy. However, q_0 and ϕ_0 are parameters independent of the proper acceleration a. The q_0 or ϕ_0 in Minkowski case Eq.(25) for $a = 0$ can be the same as the one in Rindler case Eq.(31) for $a \neq 0$, if the $\bar{\phi}(x'(\tau)) = \phi_0$ are assigned the same value in both cases. This means that a classical *point particle* has no idea about its acceleration from its monopole moment. To "know" the acceleration classically, an extended object with a finite size or a point-like detector with an outer observer co-moving at a non-zero distance is necessary. In the latter case the Planckian-like spectrum measured by the co-moving observer is simply a signal that the detector-observer pair is uniformly accelerated, just like the water-level of an accelerating bucket. Indeed, an accelerating bucket would find its water-level is oblique due to the tidal force, but if the bucket shrinks into a point, it cannot find any evidence about the acceleration by itself.

Even if the uniformly accelerated detector does not see the thermal bath according to its monopole moment, there still exist a Planck factor in the power-spectrum of the "vacuum energy", speculated by the co-moving observer with a particular choice of variable. This again suggests that the thermal interpretation may not be necessary for classical point-like detectors.

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